07-Minterm and Maxterm Expansions

Text: Unit 4

ECEGR/ISSC 201
Digital Operations and Computations
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Overview

- Minterm Expansions
- Maxterm Expansions
- Minterms and Maxterms from Switching Expressions

Minterm Expansions

- A minterm of n variables is the product of n literals in which each variable appears once in either a complemented or uncomplemented form
- F = A'BC + AB'C' + AB'C +ABC has four minterms
- F = AC is not a minterm because B is not present; we can write this expression as
  - F = AC(B'+B) = ACB' + ACB; ACB' and ACB are minterms

Minterm Expansions

- Each minterm will be equal to 1 for one combination of the variables (in this case, A,B,C)
- Consider:

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Minterm Expansions

- The minterms and associated A,B,C combinations:

A	B	C	minterms
0	0	0	A'B'C'
0	0	1	A'B'C
0	1	0	A'BC'
0	1	1	A'BC
1	0	0	AB'C'
1	0	1	AB'C
1	1	0	ABC'
1	1	1	ABC

Minterm Expansions

- For our example:
  F = A'BC + AB'C' + AB'C +ABC

Four minterms, four 1s

A	B	C	F	minterms
0	0	0	0	A'B'C'
0	0	1	0	A'B'C
0	1	0	0	A'BC'
0	1	1	1	A'BC
1	0	0	1	AB'C'
1	0	1	1	AB'C
1	1	0	0	ABC'
1	1	1	1	ABC

Minterm Notation

- There is shorthand for writing expressions that have been expanded in minterms
- Note that each row in truth table corresponds to one minterm (which may or may not be present in an expansion of F)

Minterm Notation

- If we order the truth table consistently such that the first row is always A=0, B=0, C=0, the second row is always A = 0, B =0, C =1… and the eight row is A=1, B=1, C=1 then we can correlate minterm with row number
- Example: row 4=> A=1, B =0, C=0 =>AB'C'

Minterm Notation

- F = A'BC + AB'C' + AB'C +ABC
- Also written as: F = $m_3 + m_4 + m_5 + m_7$

Row	A	B	C	F	minterms
0	0	0	0	0	A'B'C'
1	0	0	1	0	A'B'C
2	0	1	0	0	A'BC'
3	0	1	1	1	A'BC
4	1	0	0	1	AB'C'
5	1	0	1	1	AB'C
6	1	1	0	0	ABC'
7	1	1	1	1	ABC

Minterm Notation

- F = $m_3 + m_4 + m_5 + m_7$
- In more simplified form: $F(A,B,C) = \sum m(3,4,5,7)$

Row	A	B	C	F	minterms
0	0	0	0	0	A'B'C'
1	0	0	1	0	A'B'C
2	0	1	0	0	A'BC'
3	0	1	1	1	A'BC
4	1	0	0	1	AB'C'
5	1	0	1	1	AB'C
6	1	1	0	0	ABC'
7	1	1	1	1	ABC

Example

- Write the minterms of the truth table

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Example

- Write the minterms of the truth table

F = A'B'C' + AB'C' + ABC $F(A,B,C) = \sum m(0,4,7)$

Row	A	B	C	F	minterms
0	0	0	0	1	A'B'C'
1	0	0	1	0	A'B'C
2	0	1	0	0	A'BC'
3	0	1	1	0	A'BC
4	1	0	0	1	AB'C'
5	1	0	1	0	AB'C
6	1	1	0	0	ABC'
7	1	1	1	1	ABC

Maxterm Expansions

- Minterms correspond to a sum of products expression
- Maxterms correspond to a product of sums
- A maxterm of n variables is a sum of n literals in which each literal appears exactly once in true or complemented form, but not both

Maxterm Expansions

- F = (A+B+C)(A'+B'+C) has two maxterms
- F = (A+C)(A' + B' + C) is not in maxterm form
- F = (AB+C)(A'+B'+C) is not in maxterm form

Max Expansions

- Each maxterm will be equal to 0 for one combination of the variables
- From our example:
  F =(A+B+C)(A+B+C')(A+B'+C)(A'+B'+C)

Maxterm Expansions

- F =(A+B+C)(A+B+C')(A+B'+C)(A'+B'+C)

Four maxterms, four 0s

A	B	C	F	maxterms
0	0	0	0	A+B+C
0	0	1	0	A+B+C'
0	1	0	0	A+B'+C
0	1	1	1	A+B'+C'
1	0	0	1	A'+B+C
1	0	1	1	A'+B+C'
1	1	0	0	A'+B'+C
1	1	1	1	A'+B'+C'

Maxterm Notation

- $F(A,B,C) = M_0M_1M_2M_6$
- Also written as: $F(A,B,C) = \prod M(0,1,2,6)$

Row	A	B	C	F	maxterms
0	0	0	0	0	A+B+C
1	0	0	1	0	A+B+C'
2	0	1	0	0	A+B'+C
3	0	1	1	1	A+B'+C'
4	1	0	0	1	A'+B+C
5	1	0	1	1	A'+B+C'
6	1	1	0	0	A'+B'+C
7	1	1	1	1	A'+B'+C'

Minterms and Maxterms from Switching Expressions

- Can easily find minterm or maxterm from a truth table
- Truth tables can be cumbersome to create if the number of variables is large
- Algebraic way:
  - Write the expression as a sum of products and introduce missing variables using (X + X') =1 (for minterm) or XX' (for maxterm)

Minterms and Maxterms from Switching Expressions

- For example, given:
  - F = A'B' + AC
- Introduce the missing variable in each product using (X + X') = 1
  - F = A'B'(C +C') + A(B +B')C
- Expand (with reordering) and find decimal notation:
  - F = A'B'C' + A'B'C + AB'C + ABC
    (000) (001) (101) (111)
- So $F\ A,B,C = \sum m\ 0,1,5,7$

Minterms and Maxterms from Switching Expressions

- To write as Maxterm:
  - F = A'B' + AC
- Factor into product of sums (second distributive law)
  - F = (A'B' + A)(A'B' + C)
  - F = (A + A')(A+B')(C+A')(C+B')
- Using XX'=0
  - F= (A+B'+CC')(C+A' + BB')(AA'+ C+B')
- Using (X +Y)(X +Y') = X
  - F=(A+B'+C)(A+B'+C')(A'+C+B)(A'+C+B')(A+B'+C)(A'+B'+C)

Minterms and Maxterms from Switching Expressions

- Removing redundant terms:
- F=(A+B'+C)(A+B'+C')(A'+B+C)(A'+B'+C)(A+B'+C)(A'+B'+C)
- Reordering
  - F=(A+B'+C)(A+B'+C')(A'+B+C)(A'+B'+C)
- Into decimal notation (complements are 1, uncomplemented are 0):
  - 010 011 100 110
- Gives: $F\ A,B,C = \prod M(2,3,4,6)$

Minterms and Maxterms from Switching Expressions

- F = A'B' + AC
- Minterms should match F =1: $F\ A,B,C = \sum m\ 0,1,5,7$

Row	A	B	C	F	minterms	maxterms
0	0	0	0	1	A'B'C'	A+B+C
1	0	0	1	1	A'B'C	A+B+C'
2	0	1	0	0	A'BC'	A+B'+C
3	0	1	1	0	A'BC	A+B'+C'
4	1	0	0	0	AB'C'	A'+B+C
5	1	0	1	1	AB'C	A'+B+C'
6	1	1	0	0	ABC'	A'+B'+C
7	1	1	1	1	ABC	A'+B'+C'

Minterms and Maxterms from Switching Expressions

- F = A'B' + AC
- Maxterms should match F =0: $F\ A,B,C = \prod M(2,3,4,6)$

Row	A	B	C	F	minterms	maxterms
0	0	0	0	1	A'B'C'	A+B+C
1	0	0	1	1	A'B'C	A+B+C'
2	0	1	0	0	A'BC'	A+B'+C
3	0	1	1	0	A'BC	A+B'+C'
4	1	0	0	0	AB'C'	A'+B+C
5	1	0	1	1	AB'C	A'+B+C'
6	1	1	0	0	ABC'	A'+B'+C
7	1	1	1	1	ABC	A'+B'+C'

Example

- Find the minterm expansion of
  - F = A'(B'+D)+ACD'
- Basic steps:
  - Expand into SoP form (if necessary)
  - Use (X + X') =1 to add missing variables to each product
  - Multiply out
  - Remove redundant terms
  - Convert to decimal notation

Example

- F = A'(B'+D)+ACD'
- First expand into SoP form
  - F=A'B' + A'D +ACD'
- Use (X + X') =1 to add missing variables
  - F=A'B'(C + C')(D +D') + A'D(B+B')(C+C') +A(B+B')CD'
- Multiply out
- F=A'B'C'D' + A'B'C'D + A'B'CD' + A'B'CD + A'BC'D + A'BCD + ABCD' + AB'CD'

Example

- Remove redundant terms
- F=A'B'C'D' + A'B'C'D + A'B'CD' + A'B'CD + ~~A'BC'D~~ + ~~A'BCD~~ + A'BC'D + A'BCD + ABCD' + AB'CD'
- Place in decimal notation
- F=A'B'C'D' + A'B'C'D + A'B'CD' + A'B'CD + A'BC'D
  (0000) (0001) (0010) (0011) (0101)
  + A'BCD + ABCD' + AB'CD'
  (0111) (1110) (1010)

$$F\ A,B,C,D = \sum m\ 0,1,2,3,5,7,10,14$$

Example

- Check the truth table
  F = A'(B'+D)+ACD'

Row	A	B	C	D	F
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	

Row	A	B	C	D	F
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

Example

- Check the truth table
  F = A'(B'+D)+ACD' $F\ A,B,C,D = \sum m\ 0,1,2,3,5,7,10,14$

Row	A	B	C	D	F
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1

Row	A	B	C	D	F
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	0

Minterms and Maxterms from Switching Expressions

- If given the minterm expansion for F, then the maxterms are those numbers not in the minterm expansion
- If given the maxterm expansion for F, then the minterms are those numbers not in the maxterm expansion

Minterms and Maxterms from Switching Expressions

- If given the minterm expansion for F', then the minterm expansion of F are those numbers not in the minterm expansion of F'
- If given the maxterm expansion for F', then the maxterm expansion of F are those numbers not in the maxterm expansion of F'

Minterms and Maxterms from Switching Expressions

- If given the minterm expansion for F, then the maxterm expansion of F' are the same numbers as the minterms of F
- If given the maxterm expansion for F, then the minterm expansion of F' are the same numbers as the maxterms of F

Minterms and Maxterms from Switching Expressions

- If $F(A,B,C) = \prod M(2,3,4,6)$
  Then $F(A,B,C) = \sum m(0,1,5,7)$
- If $F(A,B) = \sum m(0)$
  Then $F(A,B) = \prod M(1,2,3,)$
- If $F'(A,B) = \sum m(0,3)$
  Then $F(A,B) = \sum m(1,2)$
- If $F(A,B,C) = \sum m(3,4,5,6,7)$
  Then $F'(A,B,C) = \prod M(3,4,5,6,7)$